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BUCKLING OF SANDWICH PANELS WITH A FLEXIBLE CORE—HIGH-ORDER THEORY

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Abstract—The buckling behavior of sandwich panels with a core that is flexible in the out-of-plane direction, also denoted as "soft" core including high-order effects, is presented. The buckling analysis consists of the formulation of the linear and the nonlinear governing equations along with the boundary conditions. The sandwich panel construction is general and consists of two skin-panels, metallic or composite laminated symmetric that may be unidentical and a flexible isotropic or orthotropic core made of foam or a low strength honeycomb. The analysis uses a high-order theory formulation, which permits nonlinear distortions of the cross-section plane of the core as well as changes in its height. The analysis determines the bifurcation loads along with the associated mode shapes, local or overall buckling modes, as well as deformations, internal resultants and stresses at skin–core interface layers due to imperfections. Numerical results using closed form solutions for simply-supported panels, with identical and non-identical skin-panels subjected to compressive inplane loads as well as imperfection analysis results, are presented. The results reveal that, under some structural configurations, the local buckling mode is a critical, rather than a global one as a result of the out-of-plane flexibility of the core. © 1997 Elsevier Science Ltd.

INTRODUCTION

Sandwich structures with "soft" cores made of foam or low strength honeycomb, like aramid or nomex, are being used in various industrial applications such as aerospace and civil engineering structures. The use of a foam or low strength honeycomb core, rather than a metallic honeycomb one, is advantageous in terms of weight, manufacturing processes and resources. The major difference between a metallic honeycomb and a "soft" core is due to its out-of-plane flexibility that significantly affects the overall behavior, which under various loading schemes may lead to different behavior patterns in the upper and the lower skin-panels as compared with panels whose core is infinitely stiff in the out-of-plane direction.

The instability of sandwich structures with cores made of a "stiff" metallic honeycomb has been considered by many researchers. The basic assumption used assumes that the core is antiplane and incompressible, i.e. one in which the inplane normal stresses in the longitudinal, x-direction and the transverse, y-direction, directions and the inplane shear stresses are null, the vertical shear stresses are independent of the vertical coordinate, it is incompressible in the vertical direction, and its cross-section remains plane after deformation. The general approach, in the last few decades [see Allen (1966); Plantema (1966); Zenkert (1995)], assumes that the buckling modes of the panel, i.e. the global and the local (in the form of wrinkling of the skin-panels only), are *uncoupled*. The global buckling is defined through the solution of an equivalent panel that considers the shear rigidity of the core and ignores the out of plane flexibility of the core. For local (wrinkling) buckling analysis the panel is replaced by two isolated, separate, long skin-panels resting on elastic foundation that is provided by the out of plane rigidity of the core while ignoring any interaction between the two skin-panels. A similar approach that also ignores interaction between the skin-panels and uses uncoupled buckling modes [used by Bulson (1970); Brush and Almroth (1975); Vinson (1986)]. This approach is valid as long as the core is incompressible in the vertical direction. However, when a compressible, "soft" type of core is considered, an interaction between the global and the local buckling mode exists, as well as a collaboration between the two skin-panels, thus the critical mode may shift from a global mode to a local one and vice versa. There is a group of researchers, Benson and Mayers (1967), and Pearce and Webber (1972), that used buckling modes that were symmetric and asymmetric shapes with respect to the mid-plane of the panel. In two recent papers, Hunt and Da Silva (1990a, 1990b) used an approach based on energy methods and superposition of symmetrical and asymmetrical buckling modes. This approach is limited to specific configurations and boundary conditions. A high-order theory approach, used by Kant and Patil (1991), had replaced the sandwich structure with an equivalent high-order shear deformable structure which lacks the ability to determine the local buckling modes and the imperfection effects on the overall behavior. There are numerous research works that tackle the panel stability problem numerically using: energy methods [see Whitney (1987); Kim and Hong (1988); Hassinen (1995)] or finite elements methods [see O'Conner (1987); Al-Qarra (1988); Rao and Meyer-Piening (1991)]. A different systematic rigorous approach that is based on a high-order theory and incorporates the effects, due to the out of plane flexibility of the core as well as its shear rigidity into the behavior, has been developed the author [see Frostig and Baruch (1990); Frostig *et al.* (1991, 1992b)] and has been applied to delamination and stress concentration problems [see Frostig (1992a, 1993a, 1993b)], to buckling and vibration beam behaviors [see Frostig and Baruch (1993c, 1994a)], to sandwich beams with unparalleled skins and laminated composite skins with unsymmetrical stacking sequence [see Peled and Frostig (1994, 1995); Frostig and Shenhar (1995] and to sandwich panels with a "soft" core [see Frostig and Baruch (1994a, 1994b, 1996)]. This high-order theory can deal also with unidentical boundary conditions for the upper and the lower skin-panel at the same edge. It is possible since the independent variables of the theory consist of the inplane deformations, in the longitudinal and transverse directions, the out of plane deformation as well as the rotation of *each* skin-panel separately, for more details in the case of beams and plates [see Frostig *et al.* (1993b, 1994b)]. Non-identical conditions exist whenever the supporting system of the panel with a "soft" core is imposed at the lower skin-panel only. A survey of the literature reveals that there is no systematic rigorous approach to the stability of sandwich panels with a general construction and a "soft" core and unidentical boundary conditions at the edges of the upper and the lower skin-panels.

The present analysis adopts the high-order theory approach [see Frostig (1990 to 1996); Peled (1994, 1995)] and uses variational principles to formulate the general governing non-linear behavior equations along with the appropriate boundary conditions. The governing equations at the prebuckling and the buckling stages are derived through a perturbation technique [see Simitses (1976)]. Closed form solutions are determined for some typical cases and a numerical study is conducted to define the effects of the out-of-plane flexibility of the core on the buckling behavior.

The assumptions used in the analysis are usually those encountered in elastic theories with intermediate class of deformations, i.e. small deformations with large rotation. The skins are considered as ordinary thin panels with flexural and inplane rigidities. The imperfections are small and are imposed only on the skin-panels. The core follows the assumption adopted by many researchers for a honeycomb type of core [see Allen (1966); Plantema (1966); Zenkert (1995)] i.e. it has shear resistance, but is free of inplane normal and shear stresses. This assumption is also practically correct for foam cores, since its elastic modulus and its flexural rigidity are about three and two orders, respectively, smaller than those of the skin-panels. In the case of a honeycomb core made of non-metallic materials, like aramid or nomex, this assumption is accurate [see Marshal (1982)]. The core is assumed to behave elastic, linear with small deformations while its height may change and its cross-section plane takes a nonlinear pattern after deformation.

The manuscript presents a rigorous general stability analysis. The nonlinear governing equations along with boundary conditions are derived first. The linear equations for the prebuckling and buckling stage are determined next and are followed by an imperfection buckling analysis. Closed form solutions for the pre-buckling and the buckling stages are determined for some typical sandwich constructions subjected to inplane external loads. Numerical results and a summary with conclusions are presented in the sequel.

MATHEMATICAL FORMULATION

The nonlinear governing equations and boundary conditions are derived via variational principles. Imperfections are incorporated in the analysis and the bifurcation buckling equations are derived through a perturbation procedure.

NONLINEAR ANALYSIS

The nonlinear formulation is based on a variational procedure that minimizes the functional of the total energy of the structure. The first variation of the internal and external potential energies are expressed in terms of stresses, strains, external vertical distributed loads and external inplane loads exerted at the edges of the panels.

Hence, the variation of the internal potential energy reads:

$$\delta U = \int_{V_t} (\sigma^t_{xx}\,\delta\varepsilon^t_{xx} + \sigma^t_{yy}\,\delta\varepsilon^t_{yy} + \tau^t_{xy}\,\delta\gamma^t_{xy})\,\mathrm{d}v + \int_{V_b} (\sigma^b_{xx}\,\delta\varepsilon^b_{xx} + \sigma^b_{yy}\,\delta\varepsilon^b_{yy} + \tau^b_{xy}\,\delta\gamma^b_{xy})\,\mathrm{d}v$$

$$+ \int_{V_{\mathrm{core}}} (\tau_{xz}\,\delta\gamma_{xz} + \tau_{yz}\,\delta\gamma_{yz} + \sigma_{zz}\,\delta\varepsilon_{zz})\,\mathrm{d}v \qquad (1)$$

where σ_{ii} and ε_{ii} ($i = x$ or y) are the normal stresses and strains in x- and y-directions at the upper and the lower skin panels; τ_{iz} and γ_{iz} ($i = x$ or y) are the vertical shear stresses and shear strains of the core; σ_{zz} and ε_{zz} are the normal stresses and strains in the vertical direction of the core; V_t, V_b and V_{core} are the volume of the upper and lower skin panels and the core, respectively, and dv denotes the volume of a differential segment, see Fig. 1 for coordinate and sign conventions.

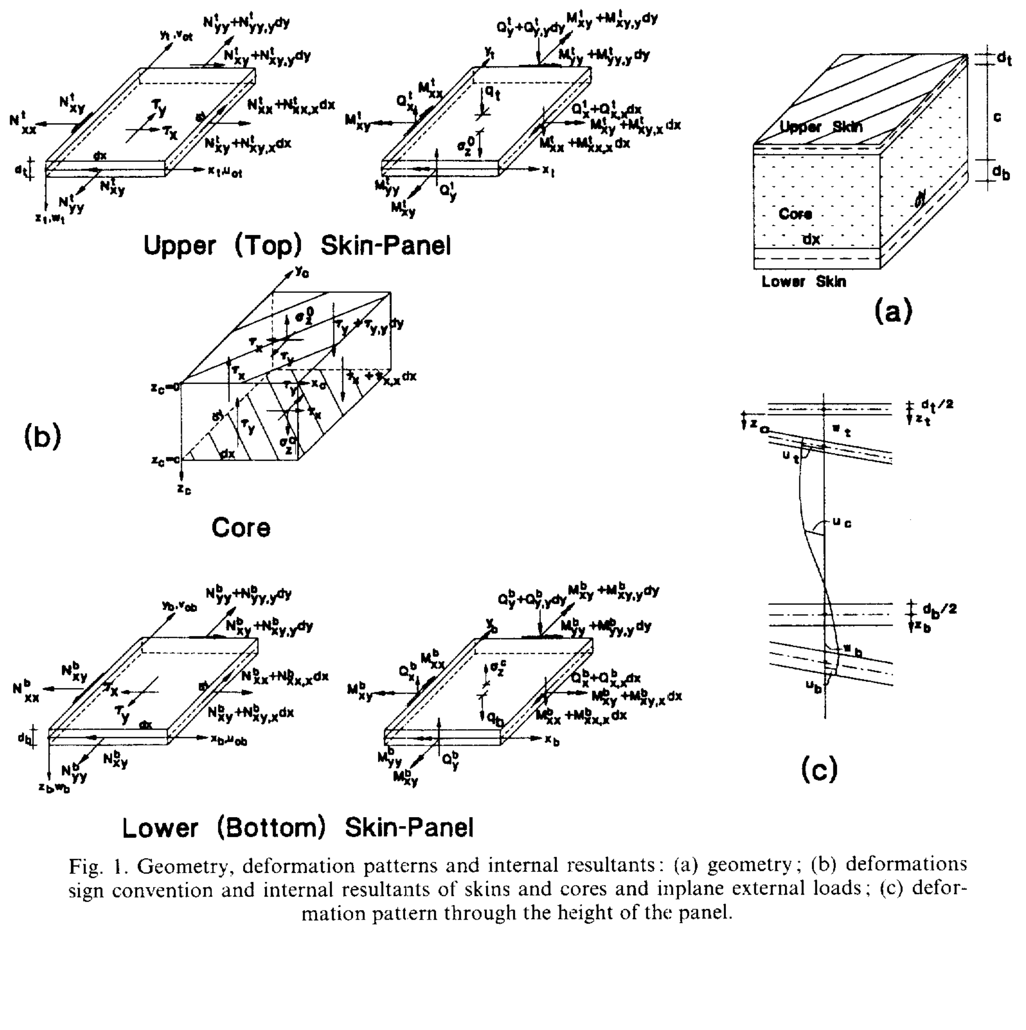

Fig. 1. Geometry, deformation patterns and internal resultants: (a) geometry; (b) deformations sign convention and internal resultants of skins and cores and inplane external loads; (c) deformation pattern through the height of the panel.

The variation of the external energy equals:

$$\delta V = -\int_0^a\int_0^b (q_t\,\delta w_t + q_b,\delta w_b)\,\mathrm{d}x\,\mathrm{d}y - \sum_{j=0}^{2}\int_C^b\int_0^a (\bar{N}_{xxtj}\,\delta u_{ot} + \bar{N}_{xytj}\,\delta v_{ot}$$
$$+\bar{N}_{xxbj}\,\delta u_{ob} + \bar{N}_{xybj}\,\delta v_{ob})\,\delta_D(x-x_j)\,\mathrm{d}x\,\mathrm{d}y$$
$$-\sum_{j=1}^{2}\int_0^b\int_0^a (\bar{N}_{yytj}\,\delta v_{ot} + \bar{N}_{xytj}\,\delta u_{ot} + \bar{N}_{yybj}\,\delta v_{ob} + \bar{N}_{xybj}\,\delta u_{ob})\,\delta_D(y-y_j)\,\mathrm{d}x\,\mathrm{d}y, \quad (2)$$

where q_t and q_b are the vertical distributed loads exerted on the upper and lower skin-panels, respectively; $\bar{N}_{xxtj}$ and $\bar{N}_{xxbj}$ are the external loads in x-direction and $\bar{N}_{xytj}$ and $\bar{N}_{xybj}$ are the external inplane shear loads applied at the upper and the lower skins-panels edges, respectively, and are exerted at $x_1 = 0$ and $x_2 = a$ with $j = 1, 2$; $\bar{N}_{yytj}$ and $\bar{N}_{yybj}$ are the external inplane loads in the y-direction and $\bar{N}_{xytj}$ and $\bar{N}_{xybj}$ are the external inplane shear loads applied at the edges of the upper and the lower skins-panels, respectively, and are exerted at $y_1 = 0$ and $y_2 = b$ with $j = 1, 2$; $\delta_d\,(x-x_j)$ and $\delta_d\,(y-y_j)$ are the Delta of Dirac functions at the location of the load: w_i, u_{oi} and v_{oi} are the vertical deflection and inplane displacements in x- and y-directions of the mid-plane of each skin-panel $i = t, b$, respectively. The second and the third terms in eqn (2), after integration with respect to the Dirac functions, actually describe the contribution of the external loads on the transverse and the longitudinal edges of the panel, respectively. Geometry and sign convention for stresses, displacements and loads appear in Fig. 1.

The kinematic relations for the skin-panels based on small deformations with large rotations of the skin-panels [see Brush and Almroth (1975)] read as follows. For each skin-panel:

$$\begin{aligned}\varepsilon_{xxi} &= \varepsilon_{xxoi} + z_i\chi_{xxi}\\ \varepsilon_{yyi} &= \varepsilon_{xxoi} + z_i\chi_{yyi} \quad (i = t, b)\\ \gamma_{xyi} &= \gamma_{xyoi} + z_i\chi_{xyi},\end{aligned} \quad (3)$$

where the mid-plane inplane strains and curvatures read

$$\begin{aligned}\varepsilon_{xxoi} &= u_{oi,x} + 1/2w_{i,x}^2 + w_{i,x}w_{i,x}^*\\ \varepsilon_{yyoi} &= v_{oi,y} + 1/2w_{i,y}^2 + w_{i,y}w_{i,y}^* \quad (i = t, b)\\ \gamma_{xyoi} &= u_{oi,y} + v_{oi,x} + w_{i,x}w_{i,y} + w_{i,x}w_{i,y}^* + w_{i,y}w_{i,x_i}^*\\ \chi_{xxi} &= -w_{i,xx} \quad \chi_{yyi} = -w_{i,yy} \quad \chi_{xyi} = -2w_{i,xy},\end{aligned} \quad (4)$$

where ε_{xxoi}, ε_{yyoi} and γ_{xyoi} $(i = t, b)$ are the inplane strains in x- and y-directions and the inplane shear angle of the mid-plane of the upper and the lower skin-panels, respectively; χ_{xxi}, χ_{yyi} and χ_{xyi} $(i = t, b)$ are the curvature in the x- and y-directions and the torsion curvature of the skin-panels, respectively; $u_{oi}(x, y)$ and $v_{oi}(x, y)$ $(i = t, b)$ are the inplane displacements in x- and y-directions of the mid-plane of each skin-panel, respectively; $w_i(x, y)$ and $w_i^*(x)$ are the vertical and imperfection deflections of each skin-panel, respectively $(i = t, b)$; z_i is the vertical coordinate of each skin-panel $(i = t, b)$ and is measured downward from the mid-plane of each skin-panel [see Fig. 1(b)] and $(\ )_{,kl}$ denotes a partial derivative with respect to k or l.

The kinematic relations of the core that are based on small deformations:

$$\begin{aligned}\gamma_{xz} &= u_{c,z} + w_{c,x}\\ \gamma_{yz} &= v_{c,z} + w_{c,y}\end{aligned}$$

$$\varepsilon_{zz} = w_{c,z}, \tag{5}$$

where $u_c(x, y, z_c)$, $v_c(x, y, z_c)$ and $w_c(x, y, z_c)$ are the x- and y-displacements and the vertical deflection of the core, respectively, and z_c is the vertical coordinate of the core, measured downward from the upper skin-core interface [see Fig. 1(b)].

The compatibility conditions, assuming full bond between the core and the skin-panels, at the upper and the lower skin–core interface, $(j = t, b)$, equal:

$$u_c(z_c = z_j) = u_{oj} + (-1)^k \frac{d_l}{2} w_{j,x}$$

$$v_c(z_c = z_j) = v_{oj} + (-1)^k \frac{d_j}{2} w_{j,v}$$

$$w_c(z_c = z_j) = w_j \tag{6}$$

where $k = 0$ when $j = t$ and $k = l$ when $j = b$; $z_t = 0$ at the upper interface and $z_b = c$ at the lower interface, $u_c(z_c = z_j)$, $v_c(z_c = z_j)$ and $w_c(z_c = z_j)$ with $z_j = 0$, c $(j = t, b)$ are the displacements in the x- and y-directions and the vertical deflection, respectively, in the core at the upper and the lower interface layers; and d_j $(j = t, b)$ and c are the thickness of the upper and the lower skin-panels and the height of the core, respectively [see Fig. 1(a) and (b)].

The field equations and the boundary conditions are derived using eqns (1) and (2), along with the kinematic relations, eqns (3)–(5), with the compatibility conditions, eqn (6), and using internal resultants [see Fig. 1(b)]. Hence, through substitution of the internal resultants in eqn (1) and after integration by parts and some algebraic manipulation, the governing equations read as follows. For the upper and the lower skin-panels $(j = t, b)$:

$$N^j_{xx,x} + N^j_{xy,y} - (-1)^k \tau_{xz}(z_c = z_j) = 0 \tag{7}$$

$$N^j_{yy,y} + N^j_{xy,x} - (-1)^k \tau_{yz}(z_c = z_j) = 0 \tag{8}$$

$$M^j_{xx,xx} + M^j_{yy,yy} + 2M^j_{xy,xy} + (-1)^k \sigma_{zz}(z_c = z_j) + (\tau_{xz,x}(z_c = z_j) + \tau_{yz,y}(z_c = z_j)) \frac{d_j}{2}$$
$$+ [N^j_{xx}(w_{j,x} + w^*_{j,x})]_{,x} + [N^j_{yy}(w_{j,y} + w^*_{j,y})]_{,y} + [N^j_{xy}(w_{j,x} + w^*_{j,x})]_{,y}$$
$$+ [N^j_{xy}(w_{j,y} + w^*_{j,y})]_{,x} = -q_j, \tag{9}$$

where N^j_{xx}, N^j_{yy} and N^j_{xy} $(j = t, b)$ are the inplane normal stress resultants in x- and y-directions and the shear stress resultant, respectively, at the upper and the lower skin panels and M^j_{xx}, M^j_{yy} and M^j_{xy} $(j = t, b)$ are the bending moment stress resultant in the x- and y-directions and the torsion moment resultant, respectively, at the upper and the lower skin-panels. For sign convention see Fig. 1(b).

The governing equations of the core read:

$$\tau_{xz,z} = 0 \tag{10}$$

$$\tau_{yz,z} = 0 \tag{11}$$

$$\tau_{xz,x} + \tau_{yz,y} + \sigma_{zz,z} = 0. \tag{12}$$

The shear stresses, τ_{xz} and τ_{yz}, see eqns (10) and (11), are uniform through the height of the core and are functions of the x and y only. Thus:

$$\tau_{xz}(x,y,z_c) = \tau_x(x,y) \quad \tau_{yz}(x,y,z_c) = \tau_y(x,y) \tag{13}$$

or τ_{xz} ($z_c = 0$ or c) $= \tau_x$ and τ_{yz} ($z_c = 0$ or c) $= \tau_y$ [see eqns (7)–(9)].

The boundary conditions for each skin-panel and core at the panel edges, at $x = 0, a$ and $y = 0, b$ and at the upper and lower skin-panels, $(j = t, b)$:

$$N^j_{xx}(x = 0 \quad \text{or } x = a) = -\bar{N}_{xxj0} \quad \text{or } \bar{N}_{xxja} \quad \text{or } u_{oj}(x = 0 \quad \text{or } x = a) = 0 \tag{14}$$

$$N^j_{xy}(x = 0 \quad \text{or } x = a) = -\bar{N}_{xyj0} \quad \text{or } \bar{N}_{xyja} \quad \text{or } v_{oj}(x = 0 \quad \text{or } x = a) = 0 \tag{15}$$

$$N^j_{yy}(y = 0 \quad \text{or } y = b) = -\bar{N}_{yyj0} \quad \text{or } \bar{N}_{yyjb} \quad \text{or } v_{oj}(y = 0 \quad \text{or } y = b) = 0 \tag{16}$$

$$N^j_{xy}(y = 0 \quad \text{or } y = b) = -\bar{N}_{xyj0} \quad \text{or } \bar{N}_{xyjb} \quad \text{or } u_{oj}(y = 0 \quad \text{or } y = b) = 0 \tag{17}$$

$$M^j_{xx}(x = 0 \quad \text{or } x = a) = 0 \quad \text{or } w_{j,x}(x = 0 \quad \text{or } x = a) = 0 \tag{18}$$

$$M^j_{yy}(y = 0 \quad \text{or } y = b) = 0 \quad \text{or } w_{j,y}(y = 0 \quad \text{or } y = b) = 0 \tag{19}$$

$$M^j_{xx,x} + 2M^j_{xy,y} + \tau_x \frac{d_j}{2} + N^j_{xx}(w_{j,x} + w^*_{j,x}) + N^j_{x}(w_{j,y} + w^*_{j,y})|(x = 0 \quad \text{or } x = a) = 0$$

$$\text{or } w_j(x = 0 \quad \text{or } x = a) = 0 \tag{20}$$

$$M^i_{yy,y} + 2M^j_{xy,x} + \tau_y \frac{d_j}{2} + N^j_{yy}(w_{j,y} + w^*_{j,y}) + N^j_{xy}(w_{j,x} + w^*_{j,x})|(y = 0 \quad \text{or } y = b) = 0$$

$$\textit{or } w_j(y = 0 \quad \text{or } y = b) = 0 \tag{21}$$

$$M^j_{xy} \quad \text{at } ((x = 0 \quad \text{or } x = a) \quad \text{and} \quad (y = 0 \quad \text{or } y = b)) = 0$$

$$\textit{or } w_j((x = 0 \quad \text{or } x = a) \quad \text{and} \quad (y = 0 \quad \text{or } y = b)) = 0, \tag{22}$$

where $\bar{N}_{xxjk}$, $\bar{N}_{yyjk}$ and $\bar{N}_{xyjk}$ ($k = 0$ or a or $b, j = t, b$) are the inplane external loads in the x- and y-directions and the shear inplane external loads exerted at the edges of the upper and the lower skin-panels, at $x = 0$ or a, and $y = 0$ or b, respectively. It should be noted that, since the conditions described by eqns (14)–(22) are imposed on the upper and lower skin-panels independently, non-identical boundary conditions are allowed at the same edge.

The conditions at the edges of the core at $z_c = z$, read:

$$\tau_x(x = 0 \quad \text{or } x = a) = 0 \quad \text{or } w_c((x = 0 \quad \text{or } x = a), z) = 0 \tag{23}$$

$$\tau_y(y = 0 \quad \text{or } y = b) = 0 \quad \text{or } w_c((y = 0 \quad \text{or } y = b), z) = 0. \tag{24}$$

It should be noticed that these conditions must be fulfilled at any point through the height of the core.

The constitutive relations for each skin-panel assuming a laminated composite elastic behavior with a symmetrical lay-up read as follows. The inplane resultants relations are:

$$\begin{aligned} N^j_{xx} &= A_{11j}\varepsilon_{xxoj} + A_{12j}\varepsilon_{yyoj} + A_{16j}\gamma_{xyoi} \\ N^j_{yy} &= A_{12j}\varepsilon_{xxoj} + A_{22j}\varepsilon_{yyoj} + A_{26j}\gamma_{xyoj} \quad (j = t, b) \\ N^j_{xy} &= A_{16j}\varepsilon_{xxoj} + A_{26j}\varepsilon_{yyoj} + A_{66j}\gamma_{xyoj}, \end{aligned} \tag{26}$$

and the bending and torsion moment resultants relations

$$M_{xx}^j = D_{11j}\chi_{xxj} + D_{12j}\chi_{yyj} + D_{16j}\chi_{xyj}$$
$$M_{yy}^j = D_{12j}\chi_{xxj} + D_{22j}\chi_{yyj} + D_{26j}\chi_{xyj} \quad (j = t, b)$$
$$M_{xy}^j = D_{16j}\chi_{xxj} + D_{26j}\chi_{yyj} + D_{66j}\chi_{xyj}, \tag{27}$$

where A_{mnj} and D_{mnj} $(m, n = 1, 2, 6, j = t, b)$ are the reduced inplane and flexural rigidities [see Whitney (1987)] of the upper and the lower skin-panel laminates, respectively.

The constitutive relations for an orthotropic antiplane compressible core are:

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E_{cv}} \quad \gamma_{xz} = \frac{\tau_{xz}}{G_{cxv}} \quad \gamma_{yz} = \frac{\tau_{xz}}{G_{cyv}}, \tag{28}$$

where G_{cxv}, G_{cyv} and $E_{cv} = E_c$ are the shear moduli of the core and its elastic modulus in the vertical direction, respectively.

In order to express the governing equations in terms of the deformations of the upper and the lower skin-panels the stress and the deformation fields of the core must be determined first. The stress and the deformation fields of the core are analytically determined using the compatibility conditions, see eqn (6) along with constitutive relation, see eqn (28) and following the procedure described in Frostig and Baruch (1994, 1996).

The vertical normal stresses and the vertical deformations through the depth of the core read:

$$\sigma_{zz}(x, y, z_c) = \frac{(\tau_{x,x} + \tau_{y,y})(2z_c - c)}{2} + \frac{(w_b - w_t)E_{cv}}{c} \tag{29}$$

$$w_c(x, y, z_c) = -\frac{(\tau_{x,x} + \tau_{y,y})(-z_c^2 + cz_c)}{2E_{cv}} + \frac{(w_b - w_t)z_c}{cE_{cv}} + w_t. \tag{30}$$

Hence, the vertical normal stresses at the upper and the lower skin–core interfaces, see eqn (9), equal:

$$\sigma_{zz}(x, y, z_c = 0) = -\frac{(\tau_{x,x} + \tau_{y,y})c}{2} + \frac{(w_b - w_t)E_{cv}}{c}$$
$$\sigma_{zz}(x, y, z_c = c) = +\frac{(\tau_{x,x} + \tau_{y,y})c}{2} + \frac{(w_b - w_t)E_{cv}}{c}. \tag{31}$$

The deformations of the core in x- and y-directions through the depth of the core, using the compatibility conditions at the upper skin only, see eqn (6):

$$u_c(x, y, z_c) = \frac{\tau_x z_c}{G_{cxv}} - \frac{(\tau_{x,xx} + \tau_{y,xy})(-2z_c^3 + 3cz_c^2)}{12E_{cv}} - \frac{(w_{b,x} - w_{t,x})z_c^2}{2c} - w_{t,x}\left(z_c + \frac{d_j}{2}\right) + u_{ot} \tag{32}$$

$$v_c(x, y, z_c)\frac{\tau_y z_c}{G_{cyv}} = \frac{\tau_{y,yy} + \tau_{x,yy})(+2z_c^3 + 3cz_c^3)}{12E_{cv}} - \frac{(w_{b,y} - w_{t,y})z_c^2}{2c} - w_{t,y}\left(z_c + \frac{d_j}{2}\right) + v_{ot}. \tag{33}$$

The deformation in the various directions have a nonlinear pattern through the depth of the core that are actually high-order effects that other theories lack or ignore. These high-order effects must be considered when "soft" cores are of concern.

The governing equations are formulated in terms of the following eight unknowns; the in-plane deformations of the mid-plane of each skin-panels, in the x-direction, u_{ot} and u_{ob}; in the y-direction, v_{ot} and v_{ob}; the vertical deflections of the upper and the lower skin-panels, w_t and w_b; and the shear stresses in the core in the vertical direction, τ_x and τ_y.

Hence, six equations (out of the eight) are determined through the substitution of the constitutive relations, see eqns (26) and (27), in the field equations, eqns (7)–(9) and use of eqns (31). The additional two equations are derived using the inplane displacement distributions of the core in x- and y-directions, see eqns (32) and (33), and the compatibility requirements at the lower skin-core interface, see eqn (6). In order to avoid lengthy equations the inplane resultants are used instead of their constitutive relation counterparts. Hence, the governing equations are:

$$N^{\mathrm{t}}_{xx,x}+N^{\mathrm{t}}_{xy,y}+\tau_x=0 \tag{34}$$

$$N^{\mathrm{t}}_{yy,y}+N^{\mathrm{t}}_{xy,x}+\tau_y=0 \tag{35}$$

$$N^{\mathrm{b}}_{xx,x}+N^{\mathrm{b}}_{xy,y}-\tau_x=0 \tag{36}$$

$$N^{\mathrm{b}}_{yy,y}+N^{\mathrm{b}}_{xy,x}-\tau_y=0 \tag{37}$$

$$\begin{aligned}-D_{11\mathrm{t}}w_{t,xxx}&-2(D_{12\mathrm{t}}+2D_{66\mathrm{t}})w_{\mathrm{t},xxyy}-4D_{16\mathrm{t}}w_{\mathrm{t},xxxy}-4D_{26\mathrm{t}}w_{\mathrm{t},xyyy}-D_{22\mathrm{t}}w_{\mathrm{t},yyyy}\\&+\frac{E_{\mathrm{cv}}(w_{\mathrm{b}}-w_{\mathrm{t}})}{c}+(\tau_{x,x}+\tau_{y,y})\frac{d_t+c}{2}-\tau_x(w_{\mathrm{t},x}+w^*_{\mathrm{t},x})-\tau_y(w_{\mathrm{t},y}+w^*_{\mathrm{t},y})\\&+N^{\mathrm{t}}_{xx}(w_{\mathrm{t},xx}+w^*_{\mathrm{t},xx})+N^{\mathrm{t}}_{yy}(w_{\mathrm{t},yy}+w^*_{\mathrm{t},yy})+2N^{\mathrm{t}}_{xy}(w_{\mathrm{t},xy}+w^*_{\mathrm{t},xy})=-q_{\mathrm{t}}\end{aligned} \tag{38}$$

$$\begin{aligned}-D_{11\mathrm{b}}w_{\mathrm{b},xxx}&-2(D_{12\mathrm{b}}+2D_{66\mathrm{b}})w_{\mathrm{b},xxyy}-4D_{16\mathrm{b}}w_{\mathrm{b},xxxy}-4D_{26\mathrm{b}}w_{\mathrm{b},xyyy}-D_{22\mathrm{b}}w_{\mathrm{b},yyyy}\\&-\frac{E_{\mathrm{cv}}(w_{\mathrm{b}}-w_{\mathrm{t}})}{c}+(\tau_{x,x}+\tau_{y,y})\frac{d_{\mathrm{b}}+c}{2}+\tau_x(w_{\mathrm{b},x}+w^*_{\mathrm{b},x})+\tau_y(w_{\mathrm{b},y}+w^*_{\mathrm{b},y})\\&+N^{\mathrm{b}}_{xx}(w_{\mathrm{b},xx}+w^*_{\mathrm{b},xx})+N^{\mathrm{b}}_{yy}(w_{\mathrm{b},yy}+w^*_{\mathrm{b},yy})+2N^{\mathrm{b}}_{xy}(w_{\mathrm{b},xy}+w^*_{\mathrm{b},xy})=-q_{\mathrm{b}}\end{aligned} \tag{39}$$

$$\frac{\tau_x c}{G_{cxv}}-\frac{(\tau_{x,xx}+\tau_{y,xy})c^3}{12E_{\mathrm{cv}}}-\frac{(c+d_{\mathrm{t}})}{2}w_{\mathrm{t},x}-\frac{(c+d_{\mathrm{b}})}{2}w_{\mathrm{b},x}+u_{\mathrm{ot}}-u_{\mathrm{ob}}=0 \tag{40}$$

$$\frac{\tau_y c}{G_{cyv}}-\frac{(\tau_{y,yy}+\tau_{y,xy})c^3}{12E_{\mathrm{cv}}}-\frac{(c+d_{\mathrm{t}})}{2}w_{\mathrm{t},y}-\frac{(c+d_{\mathrm{b}})}{2}w_{\mathrm{b},y}+v_{\mathrm{ot}}-v_{\mathrm{ob}}=0. \tag{41}$$

This set of eight nonlinear PDEs, in terms of the aforementioned unknowns, comprises six second-order equations and two fourth-order equations and, therefore, requires 20 boundary conditions, see eqns (14)–(24). These governing equations are an enhancement and a generalization of the equations used by other authors [see Allen (1966); Plantema (1966); Bulson (1970); Brush and Almroth (1975); Zenkert (1995)]. This set of equations reduces to the governing equations of sandwich beams with a "soft" core [see Frostig and Baruch (1993c)] when the terms with respect to the y-direction and inplane shear resultants are null and eqns (35), (37), (39) and (41) are ignored. A linearized solution approach, that leads to prebuckling and buckling stages, is considered since no general solution exist to this set of equations.

BIFURCATION ANALYSIS—LINEARIZED EQUATIONS

The behavior equations of the prebuckling and the buckling stages are derived through a perturbation and linearization technique [see Simitses (1976)], where the load scheme

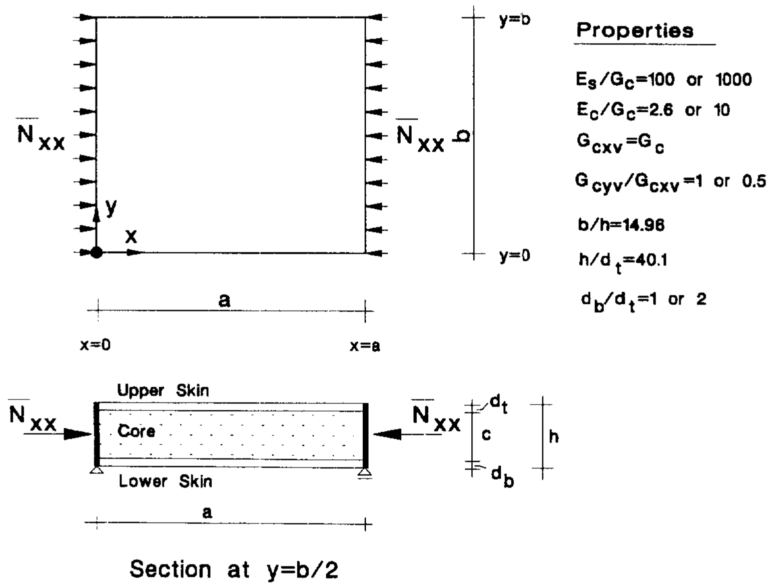

Fig. 2. Inplane loads scheme, geometrical and mechanical properties of typical sandwich panel.

consists of inplane loads applied at the panels edges only, see Figs 2 and 6, and the imperfection deformations are null. Thus, the unknowns take the following form:

$$\begin{aligned} u_{oj} &= u_{oj}^{(0)}+\zeta u_{oj}^{(1)} \quad v_{oj} = v_{oj}^{(0)}+\zeta v_{oj}^{(1)} \\ w_j &= w_j^{(0)}+\zeta w_j^{(1)} \quad (j=t,b) \\ \tau_x &= \tau_x^{(0)}+\zeta\tau_x^{(1)} \quad \tau_y = \tau_y^{(0)}+\zeta\tau_y^{(1)}, \end{aligned} \tag{42}$$

where the superscript (0) refers to the prebuckling state or a membrane state and superscript (1) refers to the buckling state and $\zeta \ll 1$ is the perturbation parameter. Hence, after the substitution of eqn (42) in the constitutive relations, eqns (26) and (27) along with the kinematic relations, eqn (4), and after linearization the stress resultants:

$$\begin{aligned} N_{mn}^j &= N_{mnj}^{(0)}+\zeta N_{mnj}^{(1)} \quad (m=x,y,n=x,y,j=t,b) \\ M_{mn}^j &= M_{mnj}^{(0)}+\zeta M_{mnj}^{(1)}, \end{aligned} \tag{43}$$

where

$$\left.\begin{aligned} N_{mnj}^{(0)} &= A_{1lj}u_{oj,x}^{(0)}+A_{l2j}v_{oj,y}^{(0)}+A_{l6j}(u_{oj,y}^{(0)}+v_{oj,x}^{(0)}) \\ N_{mnj}^{(1)} &= A_{1lj}u_{oj,x}^{(1)}+A_{l2j}v_{oj,x}^{(1)}+A_{l6j}(u_{oj,y}^{(1)}+v_{oj,x}^{(1)}) \\ M_{mnj}^{(0)} &= -D_{1lj}w_{j,xx}^{(0)}-D_{l2j}w_{j,yy}^{(0)}-2D_{l6j}w_{j,xy}^{(0)} \\ M_{mnj}^{(1)} &= -D_{1lj}w_{j,xx}^{(1)}-D_{l2j}w_{j,yy}^{(1)}-2D_{l6j}w_{j,xy}^{(1)}, \end{aligned}\right\} (m=x,y,\ n=x,y,\ l=1,2,6,\ j=t,b) \tag{44}$$

and A_{ilj} and D_{ilj} $(l=1,2,6)$ are the reduced stiffness coefficients, see eqns (26) and (27).

The bifurcation equations are derived assuming that the prebuckling stage consists of a membrane state with no vertical deformation, $w_j^{(0)}=0$ $(j=t,b)$. The governing equations, for the two stages, are derived by substituting eqns (42) and (43) in eqns (34)–(41) and collecting the terms multiplied by ζ^0 and ζ^1, separately. Higher-order terms of ζ have been neglected. Thus, the governing equations are as follows. Pre-buckling state $(w_t^{(0)}=w_b^{(0)}=0)$:

$$N^{t(0)}_{xx,x}+N^{t(0)}_{xy,y}+\tau^{(0)}_x=0 \tag{45}$$

$$N^{t(0)}_{yy,y}+N^{t(0)}_{xy,x}+\tau^{(0)}_y=0 \tag{46}$$

$$N^{b(0)}_{xx,x}+N^{b(0)}_{xy,y}-\tau^{(0)}_x=0 \tag{47}$$

$$N^{b(0)}_{yy,y}+N^{b(0)}_{xy,x}-\tau^{(0)}_y=0 \tag{48}$$

$$(\tau^{(0)}_{x,x}+\tau^{(0)}_{y,y})\frac{d_t+c}{2}=0 \tag{49}$$

$$(\tau^{(0)}_{x,x}+\tau^{(0)}_{y,y})\frac{d_b+c}{2}=0 \tag{50}$$

$$\frac{\tau^{(0)}_x c}{G_{cxv}}-\frac{(\tau^{(0)}_{x,xx}+\tau^{(0)}_{y,xy})c^3}{12E_{cv}}+u^{(0)}_{ot}-u^{(0)}_{ob}=0 \tag{51}$$

$$\frac{\tau^{(0)}_y c}{G_{cyv}}-\frac{(\tau^{(0)}_{y,yy}+\tau^{(0)}_{y,xy})c^3}{12E_{cv}}+v^{(0)}_{ot}-v^{(0)}_{ob}=0. \tag{52}$$

The boundary conditions for this stage are derived in a procedure similar to those described in eqns (14)–(24). The conditions at the edges of each skin-panels ($j=t,b$) are:

$$N^{j(0)}_{xx}(x=0 \quad \text{or}\ x=a)=-\bar{N}_{xxjo} \quad \text{or}\ \bar{N}_{xxja} \quad \text{or}\ u^{(0)}_{oj}(x=0 \quad \text{or}\ x=a)=0 \tag{53}$$

$$N^{j(0)}_{xy}(x=0 \quad \text{or}\ x=a)=-\bar{N}_{xyj0} \quad \text{or}\ \bar{N}_{xyja} \quad \text{or}\ v^{(0)}_{oj}(x=0 \quad \text{or}\ x=a)=0 \tag{54}$$

$$N^{j(0)}_{yy}(y=0 \quad \text{or}\ y=b)=-\bar{N}_{yyj0} \quad \text{or}\ \bar{N}_{yyjb} \quad \text{or}\ v^{(0)}_{oj}(y=0 \quad \text{or}\ y=b)=0 \tag{55}$$

$$N^{j(0)}_{xy}(y=0 \quad \text{or}\ y=b)=-\bar{N}_{xyj0} \quad \text{or}\ \bar{N}_{xyjb} \quad \text{or}\ u^{(0)}_{oj}(y=0 \quad \text{or}\ y=b)=0 \tag{56}$$

and at the edges of the core at $z_c=z$, equal

$$\tau^{(0)}_x(x=0 \quad \text{or}\ x=a)=0 \quad \text{or}\ w^{(0)}_c((x=0 \quad \text{or}\ x=a),z)=0 \tag{57}$$

$$\tau^{(0)}_y(y=0 \quad \text{or}\ y=b)=0 \quad \text{or}\ w^{(0)}_c((y=0 \quad \text{or}\ y=b),z)=0. \tag{58}$$

The governing equations for the buckling state after linearization and neglect of the high-order terms with respect to ζ are:

$$A_{11t}u^{(1)}_{ot,xx}+2A_{16t}u^{(1)}_{ot,xy}+A_{66t}u^{(1)}_{ot,yy}+A_{26t}v^{(1)}_{ot,yy}+(A_{66t}+A_{12t})v^{(1)}_{ot,xy}+A_{16t}v^{(1)}_{ot,xx}+\tau^{(1)}_x=0 \tag{59}$$

$$A_{16t}u^{(1)}_{ot,xx}+(A_{66t}+A_{12t})u^{(1)}_{ot,xy}+A_{26t}u^{(1)}_{ot,yy}+A_{22t}v^{(1)}_{ot,yy}+2A_{26t}v^{(1)}_{ot,xy}+A_{66t}v^{(1)}_{ot,xx}+\tau^{(1)}_y=0 \tag{60}$$

$$A_{11b}u^{(1)}_{ob,xx}+2A_{16b}u^{(1)}_{ob,xy}+A_{66b}u^{(1)}_{ob,yy}+A_{26b}v^{(1)}_{ob,yy}+(A_{66b}+A_{12b})v^{(1)}_{ob,xy}+A_{16b}v^{(1)}_{ob,xx}-\tau^{(1)}_x=0 \tag{61}$$

$$A_{16b}u^{(1)}_{ob,xx}+(A_{66b}+A_{12b})u^{(1)}_{ob,xy}+A_{26b}u^{(1)}_{ob,yy}+A_{22b}v^{(1)}_{ob,yy}+2A_{26b}v^{(1)}_{ob,xy}+A_{66b}v^{(1)}_{ob,xx}-\tau^{(1)}_y=0 \tag{62}$$

$$-D_{11t}w^{(1)}_{t,xxxx}-2(D_{12t}+2D_{66t})w^{(1)}_{t,xxyy}-4D_{16t}w^{(1)}_{t,xxxy}-4D_{26t}w^{(1)}_{t,xyyy}-D_{22t}w^{(1)}_{t,yyyy}$$
$$+\frac{E_{cv}(w^{(1)}_{b}-w^{(1)}_{t})}{c}+(\tau^{(1)}_{x,x}+\tau^{(1)}_{y,y})\frac{d_t+c}{2}-\tau^{(0)}_{x}w^{(1)}_{t,x}-\tau^{(0)}_{y}w^{(1)}_{t,y}$$
$$+N^{t(0)}_{xx}w^{(1)}_{t,xx}+N^{t(0)}_{t,yy}w^{(1)}_{t,yy}+2N^{t(0)}_{xy}w^{(1)}_{t,xy}=0 \quad (63)$$

$$-D_{11b}w^{(1)}_{b,xxx}-2(D_{12b}+2D_{66b})w^{(1)}_{b,xxyy}-4D_{16b}w^{(1)}_{b,xxxy}-4D_{26b}w^{(1)}_{b,xyyy}-D_{22b}w^{(1)}_{b,yyyy}$$
$$-\frac{E_{cv}(w^{(1)}_{b}-w^{(1)}_{t})}{c}+(\tau^{(1)}_{x,x}+\tau^{(1)}_{y,y})\frac{d_b+c}{2}+\tau^{(0)}_{x}w^{(1)}_{b,x}+\tau^{(0)}_{y}w^{(1)}_{b,y}$$
$$+N^{b(0)}_{xx}w^{(1)}_{b,xx}+N^{b(0)}_{yy}w^{(1)}_{b,yy}+2N^{b(0)}_{xy}w^{(1)}_{b,xy}=0 \quad (64)$$

$$\frac{\tau^{(1)}_{x}c}{G_{cxv}}-\frac{(\tau^{(1)}_{x,xx}+\tau^{(1)}_{y,xy})c^3}{12E_{cv}}-\frac{(c+d_t)}{2}w^{(1)}_{t,x}-\frac{(c+d_b)}{2}w^{(1)}_{b,x}+u^{(1)}_{ot}-u^{(1)}_{ob}=0 \quad (65)$$

$$\frac{\tau^{(1)}_{y}c}{G_{cyv}}-\frac{(\tau^{(1)}_{y,yy}+\tau^{(1)}_{y,xy})c^3}{12E_{cv}}-\frac{(c+d_t)}{2}w^{(1)}_{t,y}-\frac{(c+d_b)}{2}w^{(1)}_{b,y}+v^{(1)}_{ot}-v^{(1)}_{ob}=0. \quad (66)$$

The appropriate boundary conditions for the buckling stage are as follows. At the edges of the upper and the lower skin-panels, $(j = t, b)$:

$$N^{j(1)}_{xx}(x=0 \quad \text{or } x=a)=0 \quad \text{or } u^{(1)}_{oj}(x=0 \quad \text{or } x=a)=0 \quad (67)$$

$$N^{j(1)}_{xy}(x=0 \quad \text{or } x=a)=0 \quad \text{or } v^{(1)}_{oj}(x=0 \quad \text{or } x=a)=0 \quad (68)$$

$$N^{j(1)}_{yy}(y=0 \quad \text{or } y=b)=0 \quad \text{or } v^{(1)}_{oj}(y=0 \quad \text{or } y=b)=0 \quad (69)$$

$$N^{j(1)}_{xy}(y=0 \quad \text{or } y=b)=0 \quad \text{or } u^{(1)}_{oj}(y=0 \quad \text{or } y=b)=0 \quad (70)$$

$$M^{j(1)}_{xx}(x=0 \quad \text{or } x=a)=0 \quad \text{or } w^{(1)}_{j,x}(x=0 \quad \text{or } x=a)=0 \quad (71)$$

$$M^{j(1)}_{yy}(y=0 \quad \text{or } y=b)=0 \quad \text{or } w^{(1)}_{j,y}(y=0 \quad \text{or } y=b)=0 \quad (72)$$

$$M^{j(1)}_{xx,x}+2M^{j(1)}_{xy,y}+\tau^{(1)}_{x}\frac{d_j}{2}+N^{j(0)}_{xx}w^{(1)}_{j,x}+N^{j(0)}_{xy}w^{(1)}_{j,y}|(x=0 \quad \text{or } x=a)=0$$
$$\text{or } w^{(1)}_{j}(x=0 \quad \text{or } x=a)=0 \quad (73)$$

$$M^{j(1)}_{yy,y}+2M^{j(1)}_{xy,x}+\tau^{(1)}_{y}\frac{d_j}{2}+N^{j(0)}_{yy}w^{(1)}_{j,y}+N^{j(0)}_{xy}w^{(1)}_{j,x}|(y=0 \quad \text{or } y=b)=0$$
$$\text{or } w^{(1)}_{j}(y=0 \quad \text{or } y=b)=0 \quad (74)$$

$$M^{j(1)}_{xy} \quad \text{at } ((x=0 \quad \text{or } x=a) \quad \text{and} \quad (y=0 \quad \text{or } y=b))=0$$
$$\text{or } w^{(1)}_{j}((x=0 \quad \text{or } x=a) \quad \text{and} \quad (y=0 \quad \text{or } y=b))=0. \quad (75)$$

At the edges of the core at $z_c = z$, they read:

$$\tau_x^{(1)}(x=0 \quad \text{or } x=a)=0 \quad \text{or } w_c^{(1)}((x=0 \quad \text{or } x=a),z)=0 \tag{76}$$

$$\tau_y^{(1)}(y=0 \quad \text{or } y=b)=0 \quad \text{or } w_c^{(1)}((y=0 \quad \text{or } y=b),z)=0. \tag{77}$$

IMPERFECTION ANALYSIS

The imperfection analysis uses the nonlinear governing equations, eqns (34)–(37), but with linearized inplane strains, eqn (4), and the linearized boundary conditions, eqns (14)–(24), and assumes that the vertical imperfection displacement, w_j^* ($j = t, b$), of the skin-panels is small, but with moderate rotation. The derivation procedure of the governing equations follows the steps described before and is based on the assumption, commonly used by many researchers [see Bulson (1970); Brush and Almroth (1975); Simitses (1976)] that the prebuckling stage is unaffected by the imperfections. Hence, the inplane resultants are those of the prebuckling stage, see eqns (45)–(52). The first four equations, eqns (34)–(37) and the last two equations, eqns (40) and (41), remain unchanged, but with linearized inplane strains. The differences are in the two equations which describe the equilibrium in the vertical direction, eqns (38) and (39), hence:

$$\begin{aligned}
&-D_{11t}w_{t,xxxx}-2(D_{12t}+2D_{66t})w_{t,xxyy}-4D_{16t}w_{t,xxyy}-4D_{26t}w_{t,xyyy}-D_{22t}w_{t,yyyy}\\
&\quad+\frac{E_{cv}(w_b-w_t)}{c}+(\tau_{x,x}+\tau_{y,y})\frac{d_t+c}{2}-\tau_x^{(0)}w_{t,x}-\tau_{t,y}^{(0)}+N_{xx}^{t(0)}w_{t,xx}+N_{yy}^{t(0)}w_{t,yy}\\
&\quad+2N_{xy}^{t(0)}w_{t,xy}=-q_t+\tau_x^{(0)}w_{t,x}^*+\tau_y^{(0)}w_{t,y}^*-N_{xx}^{t(0)}w_{t,xx}^*-N_{yy}^{t(0)}w_{t,yy}^*-2N_{xy}^{t(0)}w_{t,xy}^*
\end{aligned} \tag{78}$$

$$\begin{aligned}
&-D_{11b}w_{b,xxx}-2(D_{12b}+2D_{66b})w_{b,xxyy}-4D_{16b}w_{b,xxxy}-4D_{26b}w_{b,xyyy}-D_{22b}w_{b,yyyy}\\
&\quad-\frac{E_{cv}(w_b-w_t)}{c}+(\tau_{x,x}+\tau_{y,y})\frac{d_b+c}{2}+\tau_x^{(0)}w_{b,x}+\tau_y^{(0)}w_{b,y}+N_{xx}^{b(0)}w_{b,xx}+N_{yy}^{b(0)}w_{b,yy}\\
&\quad+2N_{xy}^{b(0)}w_{b,xy}=-q_b-\tau_x^{(0)}w_{b,x}^*-\tau_y^{(0)}w_{b,y}^*-N_{xx}^{b(0)}w_{b,xx}^*-N_{yy}^{b(0)}w_{b,yy}^*-2N_{xy}^{b(0)}w_{b,xy}^*,
\end{aligned} \tag{79}$$

where the superscript (0) refers to the inplane stress resultant of the prebuckling stage, see eqns (45)–(52).

The boundary conditions follows the ones described in eqns (14)–(24), but with differences in eqns (20) and (21) which take the following form:

$$\begin{aligned}
&M_{xx,x}^j+2M_{xy,y}^j+\tau_x\frac{d_j}{2}+N_{xx}^{j(0)}(w_{j,x}+w_{j,x}^*)+N_{xy}^{j(0)}(w_{j,y}+w_{j,y}^*)|(x=0 \quad \text{or } x=a)=0\\
&\qquad\qquad\text{or } w_j(x=0 \quad \text{or } x=a)=0
\end{aligned} \tag{80}$$

$$\begin{aligned}
&M_{yy,y}^j+2M_{xy,x}^j+\tau_y\frac{d_j}{2}+N_{yy}^{j(0)}(w_{j,y}+w_{j,y}^*)+N_{xy}^{j(0)}(w_{j,x}+w_{j,x}^*)|(y=0 \quad \text{or } y=b)=0\\
&\qquad\qquad or\ w_j(y=0 \quad \text{or } y=b)=0.
\end{aligned} \tag{81}$$

The solution of this set of nonlinear governing equations in general is very complicated. However, for some particular boundary conditions and particular loading schemes an explicit solution exists.

BIFURCATION ANALYSIS—CLOSED FORM SOLUTIONS

The governing equations of the buckling stage, see eqns (59)–(66), are linear and depend on the inplane resultants and shear stresses of the prebuckling stage, see eqns (45)–(52). A closed form solution of the prebuckling stage equations for a general laminated composite skin-panels construction is impossible. However, for some typical cases described next closed form solutions exist.

Prebuckling stage

A closed form solution of the prebuckling stage exist for isotropic and orthotropic cores with movable and immovable inplane boundary conditions. The inplane loads are applied at the edges of the panel and through its height, see Fig. 2. The distribution of the load between the two skin-panels is determined assuming that a uniform edge displacement through the height of the panel occurs, e.g. the inplane normal strains or shear angle strain in the upper and the lower skins are identical.

A closed form solution that is independent of the coordinates with movable boundary conditions exists in case of skin-panels made of laminates that consist of either a quasi-isotropic or a specially orthotropic stacking sequence as long as the poisson ratios of the two skin-panels are identical in the x-direction and in the y-directions. Thus, the inplane resultants in the upper and the lower skin-panels, $j = t, b$, are:

$$\bar{N}_{xxj0} = \bar{N}_{xxja} = \frac{\alpha_j}{\alpha_t+\alpha_b}\bar{N}_{xx}, \quad \bar{N} = {}_{yyj0} = \bar{N}_{yyjb} = \frac{\beta_j}{\beta_t+\beta_b}\bar{N}_{yy}, \quad \bar{N}_{xyj0} = \bar{N}_{xyja} = \frac{\lambda_j}{\lambda_t+\lambda_b}\bar{N}_{xy}$$

$$\tau_x^{(0)} = 0 \quad \tau_y^{(0)} = 0, \tag{82}$$

where $\bar{N}_{xx}$, $\bar{N}_{yy}$ and $\bar{N}_{xy}$ are the inplane external loads applied at the edges of the panel through its height and the distribution coefficients equal:

$$\alpha_j = \frac{A_{22j}}{A_{11j}A_{22j}-A_{12j}^2}, \quad \beta_j = \frac{A_{11j}}{A_{11j}A_{22j}-A_{12j}^2}, \quad \lambda_j = A_{66j} \quad (j = t, b), \tag{83}$$

where the inplane rigidities of the upper and the lower skin-panels for the various stacking sequence $(j = t, b)$

quasi-isotropic

$$v_t = v_b = v, \quad A_{11j} = A_{22j} = E_j d_j/(1-v^2), \quad A_{12j} = vA_{11j}, \quad A_{66j} = G_{xyj}d_j = E/(2(1+v))d_j$$

specially orthotropic

$$v_{xt} = v_{xb} = v_x, \quad v_{yt} = v_{yb} = v_y, \quad A_{11j} = E_{xj}d_j/(1-v_xv_y), \quad A_{22j} = E_{yj}d_j/(1-v_xv_y),$$
$$A_{12j} = v_xA_{11j} = v_yA_{22j}, \quad A_{66j} = G_{xyj}d_j$$

and $\bar{N}_{xxo}$, $\bar{N}_{yyo}$ and $\bar{N}_{xyo}$ are the inplane external load per unit length applied at the edges of the panel through its height. It should be noted that an inplane load applied in one direction does not yield inplane resultants in the other direction as well as shear resultants and vice versa.

In the case of a symmetrical laminated composite skin-panel, with immovable inplane conditions at the unloaded edges and at the loaded edge in the direction perpendicular to the applied load, a closed form solution exists. In this case the vertical shear stresses of the core are null, $\tau_x^{(0)} = \tau_{yx}^{(0)} = 0$, and the inplane stress resultants are independent of the coordinates and they are determined through the solution of the following set of 12 algebraic equations:

$$\bar{N}_{xxt0} = A_{11t}\varepsilon_{xxot}+A_{12t}\varepsilon_{yyot}+A_{16t}\gamma_{xyot}, \quad \bar{N}_{xxb0} = A_{11b}\varepsilon_{xxob}+A_{12t}\varepsilon_{yyob}+A_{16t}\gamma_{xyob}$$
$$\bar{N}_{yyt0} = A_{12t}\varepsilon_{xxot}+A_{22t}\varepsilon_{yyot}+A_{26t}\gamma_{xyot}, \quad \bar{N}_{yyb0} = A_{12b}\varepsilon_{xxob}+A_{22b}\varepsilon_{yyob}+A_{26t}\gamma_{xyob}$$
$$\bar{N}_{xyt0} = A_{16t}\varepsilon_{xxot}+A_{26t}\varepsilon_{yyot}+A_{66t}\gamma_{xyot}, \quad \bar{N}_{xyb0} = A_{16b}\varepsilon_{xxob}+A_{26b}\varepsilon_{yyob}+A_{66t}\gamma_{xyob}$$
$$\varepsilon_{xxot} = \varepsilon_{xxob}, \quad \varepsilon_{yyot} = \varepsilon_{yyob}, \quad \gamma_{xyot} = \gamma_{xyob} \tag{84}$$

$$\bar{N}_{xxt0}+\bar{N}_{xxb0} = \bar{N}_{xx}$$
$$\bar{N}_{yyt0}+\bar{N}_{yyb0} = \bar{N}_{yy}$$
$$\bar{N}_{xyt0}+\bar{N}_{xyb0} = \bar{N}_{xy},$$

where the unknowns are: the inplane resultants in the various skins, $\bar{N}_{xxj0}$, $\bar{N}_{yyj0}$ and $\bar{N}_{xyj0}$ ($j = t, b$), and the inplane normal and shear angle strains, ε_{xxoj}, ε_{yyoj} and γ_{xyoj} ($j = t, b$). The closed form solutions are very lengthy and their explicit description is omitted for brevity.

The general description of the inplane resultants are:

$$\begin{aligned} N_{xxj}^{(0)} &= n_{xxjxx}\bar{N}_{xx}+n_{yyjxx}\bar{N}_{yy}+n_{xyjxx}\bar{N}_{xy} \\ N_{yyj}^{(0)} &= N_{xxjyy}\bar{N}_{xx}+n_{yyjyy}\bar{N}_{yy}+n_{xyjyy}\bar{N}_{xy} \quad (j = t, b) \\ N_{xyj}^{(0)} &= n_{xxjxy}\bar{N}_{xx}+n_{yyjxy}\bar{N}_{yy}+n_{xyjxy}\bar{N}_{xy}, \end{aligned} \tag{85}$$

where n_{xxjkl}, n_{xyjkl} and n_{xyjkl} ($j = t, b$ and $k = x$ or y and $l = y$ or x) are the distribution factors of the upper and the lower skins for the inplane resultants in x- and y-directions and the shear resultant due to various inplane loads, respectively.

Buckling stage—simply-supported panel

A closed form solution of the governing equations of the buckling stage exist in the case of a panel with simply-supported edges and normal inplane loads applied at the edges. Thus, the unknowns are:

$$\begin{aligned} w_t^{(1)}(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{wtmn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, w_b^{(1)}(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{wbmn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \\ u_{ot}^{(1)}(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{utmn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, u_{ob}^{(1)}(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{ubmn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \\ v_{ot}^{(1)}(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{vtmn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\, v_{ob}^{(1)}(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{vbmn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \\ \tau_x^{(1)}(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{\tau xmn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, \tau_y^{(1)}(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{\tau ymn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}. \end{aligned} \tag{86}$$

The substitution of the unknowns, eqn (86), into the governing equations of the buckling stage, eqns (59)–(66), yields:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(\mathbf{K}_{mn}-P\mathbf{G}_{mn})\mathbf{C}_{mn} = 0, \tag{87}$$

where $\mathbf{K}_{mn}$ and $\mathbf{G}_{mn}$ are the stiffness and the geometrical matrices, respectively, and $\mathbf{C}_{mn}$ is the vector of the constants unknowns of the m, n terms and P is the eigenvalue, i.e. the external inplane load. The stiffness matrix takes the following form:

$$\mathbf{K}_{mn} = \begin{bmatrix} K_{11mn} & K_{12mn} & 0 & 0 & 0 & 0 & K_{17mn} & 0 \\ K_{12mn} & K_{22mn} & 0 & 0 & 0 & 0 & 0 & K_{28mn} \\ 0 & 0 & K_{33mn} & K_{34mn} & 0 & 0 & K_{37mn} & 0 \\ 0 & 0 & K_{34mn} & K_{44mn} & 0 & 0 & 0 & K_{48mn} \\ 0 & 0 & 0 & 0 & K_{55mn} & K_{56mn} & K_{57mn} & K_{58mn} \\ 0 & 0 & 0 & 0 & K_{56mn} & K_{66mn} & K_{67mn} & K_{68mn} \\ K_{17mn} & 0 & K_{37mn} & 0 & K_{57mn} & K_{67mn} & K_{77mn} & K_{78mn} \\ 0 & K_{28mn} & 0 & K_{48mn} & K_{58mn} & K_{68mn} & K_{78mn} & K_{88mn} \end{bmatrix} \tag{88}$$

where the explicit description of the non-zero terms of the matrix appear in appendix A.

The geometrical matrix is a diagonal matrix that reads:

$$\mathbf{G}_{mn} = \mathrm{diag}\left[0, 0, 0, 0, \pi^2\left(\frac{m^2}{a^2} n_{yx\mathrm{t}xx} + \frac{n^2}{b^2} n_{yy\mathrm{t}yy}\right)\frac{\bar{N}_{yy}}{N_{xx}} + \pi^2\left(\frac{m^2}{a^2} n_{xx\mathrm{t}xx} + \frac{n^2}{b^2} n_{xx\mathrm{t}yy}\right),\right.$$
$$\left.\pi^2\left(\frac{m^2}{a^2} n_{yy\mathrm{b}xx} + \frac{n^2}{b^2} n_{yy\mathrm{bt}yy}\right)\frac{\bar{N}_{yy}}{N_{xx}} + \pi^2\left(\frac{m^2}{a^2} n_{xx\mathrm{b}xx} + \frac{n^2}{b^2} n_{xx\mathrm{b}yy}\right), 0, 0\right]. \tag{89}$$

The vector of the unknowns is:

$$\mathbf{C}_{mn} = [C_{utmn}, C_{vtmn}, C_{ubmn}, C_{vbmn}, C_{wtmn}, C_{wbmn}, C_{\tau ymn}, C_{\tau xmn}]. \tag{90}$$

The bifurcation load is determined by solving the eigenvalue problem, see eqn (87), for each m and n terms and it equals the smallest eigenvalue among all m and n values. It should be noted that the geometrical matrix has only two non-zero terms on its diagonal, which means that for each m and n value there are only two eigenvalues with two eigenmodes. The eigenvalues can be explicitly determined, but due to their lengthy equations only numerical results for some particular cases are presented.

In the case of imperfections, with shape functions similar to those of the eigenmodes for w_t and w_b, see the first row in eqn (86), and for a given inplane load, the governing equations, eqns (34)–(37), (78)–(79) and eqns (40)–(41), take the following matrix form:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(\mathbf{K}_{mn} - P\mathbf{G}_{mn})\mathbf{C}_{mn} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\mathbf{R}_{mn}, \tag{91}$$

where the transpose vector of the right side reads:

$$\mathbf{R}_{mn} = \left[0, 0, 0, 0, \pi^2\left(\frac{m^2}{a^2} n_{yy\mathrm{t}xx} + \frac{n^2}{b^2} n_{yy\mathrm{t}yy}\right)\frac{\bar{N}_{yy}}{N_{xx}} + \pi^2\left(\frac{m^2}{a^2} n_{xx\mathrm{t}xx} + \frac{n^2}{b^2} n_{xx\mathrm{t}yy}\right),\right.$$
$$\left.\pi^2\left(\frac{m^2}{a^2} n_{yy\mathrm{b}xx} + \frac{n^2}{b^2} n_{yy\mathrm{b}yy}\right)\frac{\bar{N}_{yy}}{N_{xx}} + \pi^2\left(\frac{m^2}{a^2} n_{xx\mathrm{b}xx} + \frac{n^2}{b^2} n_{xx\mathrm{b}yy}\right), 0, 0\right]^{\mathrm{T}}. \tag{92}$$

For brevity, only numerical results for some cases are presented.

NUMERICAL STUDY

The numerical results include bifurcation analysis of sandwich panels with isotropic and orthotropic "soft" core and quasi-isotropic identical and non-identical skin-panel laminates and are merely included to describe the ability of the proposed analysis rather than a parametric study.

Bifurcation buckling

The bifurcation load corresponds to the smallest eigenvalue of eqn (87) and depends on the geometry and the mechanical properties of the skin-panels and the core. It also depends heavily on the modulus of elasticity of the core in the vertical direction, E_{cv}, and the shear moduli of the core, G_{cxv} and G_{cyv}. The results are presented in terms of the bifurcation coefficient, $K_{cr} = \bar{N}_{xx}(b^2/\pi^2 D^2)$, vs the panel aspect ratio, a/b. The effects of the core properties on the critical loads have been investigated for "soft" cores with $E/G_{cxv} = 100$ and 1000 and for a "stiff" one with $E_s/G_{cxv} = 10$ where $E_c/G_{cxv} = 2(1+v_c) = 2.6$. The results based on the classical theories for global buckling and wrinkling [see Allen (1966); Plantema (1966); Bulson (1970); Column Research Committee of Japan (1976); Zenkert (1995)] are presented for comparison.

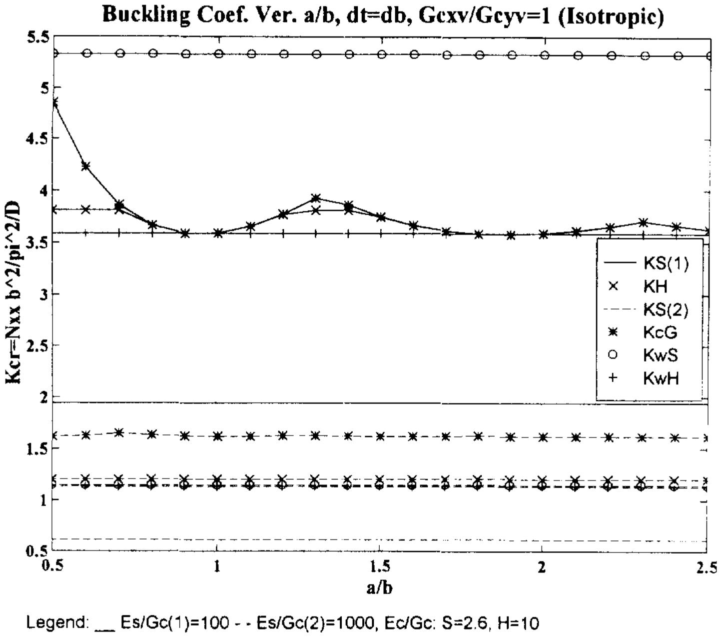

Fig. 3. Critical buckling load ratio, K_{cr} vs panel aspect ratio of a simply-supported panel with isotropic skin-panels, $d_t = d_b$, and an isotropic core.

The geometry of the investigated simply-supported panel along with the external load, $\bar{N}_{xx}$ appear in Fig. 2. The cases discussed include: identical and nonidentical with $d_b = 2d_t$ isotropic skin-panels and an isotropic core; and identical isotropic skin-panels with an orthotropic of the core, $G_{cyv}/G_{cxv} = 0.5$.

The results of the sandwich panel with the isotropic identical skin-panels and with an isotropic core appear on Fig. 3. The results include the critical loads determined by the proposed analysis and the global and wrinkling critical loads determined by the classical theories. The results of the proposed theory in the case of a "soft" core are smaller than those of the classical global and wrinkling loads while in the case of a "stiff" core, $E_c/G_{cxv} = 10$, the results of the proposed and the classical theories (kcG) coincide. In the case of "soft" core, where $E_s/G_c = 100$, described by the solid line (kS(1)) curves the critical load is independent of the aspect ratio of the panel and the buckling mode corresponds to $w_{tmax}/w_{bmax} = -1$, where w_{tmax} and w_{bmax} are extreme amplitudes of the deflection mode of the upper and the lower skin-panels, respectively. It means that each skin-panel is displaced opposite to the other. However, in the case of a "stiff" core (kH) the buckling mode shifts to a global one with $w_{tmax}/w_{bmax} = 1$, which means that the two skins are displaced in the same direction. In the case of $E_s/G_c = 1000$, described by the dash line curves (kS(2)), the loads are independent of the panel aspect ratio and the buckling mode corresponds to $w_{tmax}/w_{bmax} = -1$ with very high mode numbers. The classical results, global and wrinkling critical loads (kwS, kwH), in this case yields higher values for the buckling load then those predicted by the proposed theory.

The second case deals with unidentical isotropic skin-panels, where $d_b = 2d_t$, and the core is isotropic. The results appear on Fig. 4. The results of the high-order analysis and those of the classical ones coincide when the core is stiff. In the cases of a "soft" core the results either coincide with the wrinkling predictions based on classical approaches or are lower.

In cases of an orthotropic core with $G_{cyv}/G_{cxv} = 0.5$ and the skin-panels are isotropic and identical the critical load ratio appear on Fig. 5. In this case the results of the proposed theory are always lower as compared with the classical ones. In the case of $E_s/G_c = 100$ and $E_c/G_c = 10$ the global buckling mode governs, thus $w_{bmax}/w_{tmax} = 1$. It should be noticed that, in the case of a "soft" core, the load level has been drastically reduced as compared with the "stiff" core and the buckling mode shifts from the global mode to the local one

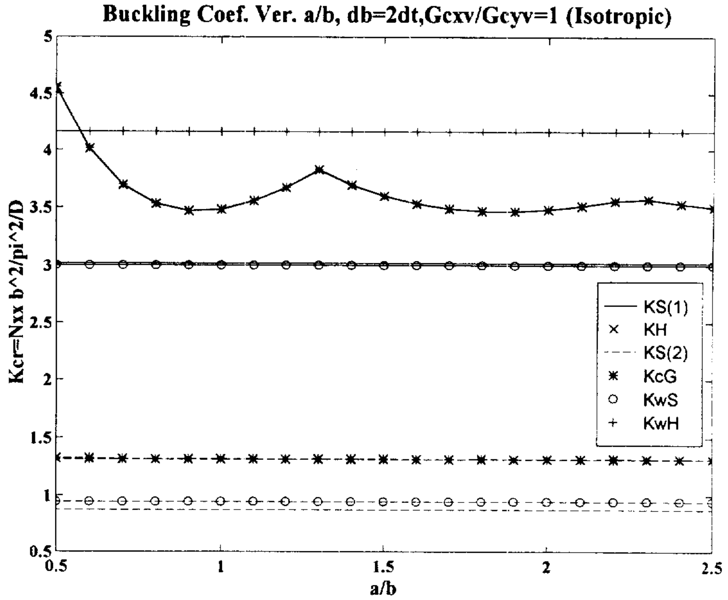

Fig. 4. Critical buckling load ratio, K_{cr} vs panel aspect ratio of a simply-supported panel with isotropic skin-panels with $d_b = 2d_t$ and an isotropic core.

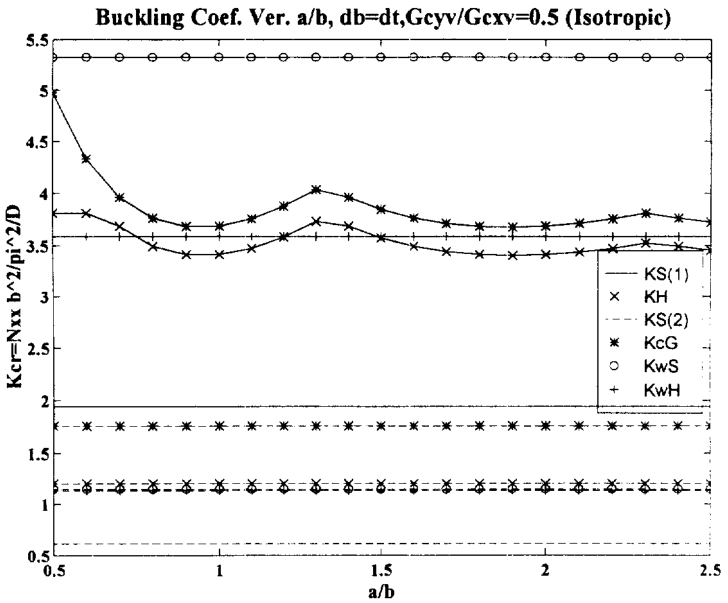

Fig. 5. Critical buckling load ratio, K_{cr} vs panel aspect ratio of a simply-supported panel with isotropic skin-panels, $d_t = d_b$, and an orthotropic core, $G_{cyv} = 0.5G_{cxv}$.

with $w_{bmax}/w_{tmax} = -1$ and with high mode numbers. In the other case, where $E_s/G_c = 1000$, a local buckling mode governs.

Imperfection analysis

The behavior of a panel subjected to simultaneous compressive inplane loads at its x and y edges with symmetrical and unsymmetrical imperfection pattern has been studied. The numerical study is used to reveal the pace at which the extreme values of the deflections, stress resultants and the peeling stresses increase as the external loads approach the critical

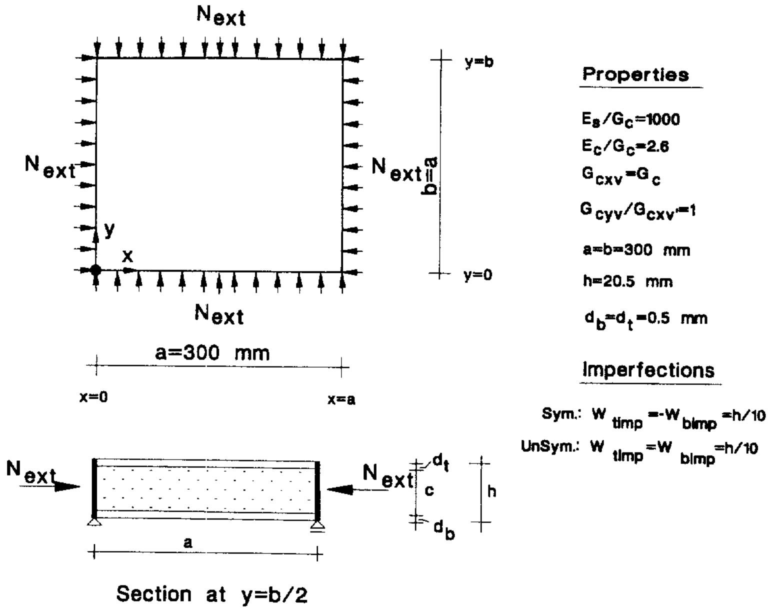

Fig. 6. Simultaneous inplane external loads, geometrical and mechanical properties of a sandwich panel with imperfections.

load of the panel. The loading scheme and the geometrical and mechanical properties of the panel appear in Fig. 6. The imperfection pattern chosen equals:

$$w_{\text{timp}}(x,y) = W_{\text{timp}} \sin\frac{m_{\text{imp}}\pi x}{a} \sin\frac{n_{\text{imp}}\pi y}{b}, \quad w_{\text{bimp}}(x,y) = W_{\text{bimp}} \sin\frac{m_{\text{imp}}\pi x}{a} \sin\frac{n_{\text{imp}}\pi y}{b}, \tag{93}$$

where $m_{\text{imp}} = n_{\text{imp}} = 27$ and the symmetrical and unsymmetrical imperfection amplitudes, with respect to the midheight of the panel, read as the symmetrical mode: $W_{\text{timp}} = -W_{\text{bimp}} = h/10$ and the unsymmetrical mode: $W_{\text{timp}} = W_{\text{bimp}} = h/10$, where h is the total height of panel.

The results, see Fig. 7, include the extreme values of the vertical deflections and the bending moments of the upper and the lower skin-panels, the shear resultants of the skin-panels and the core, and the peeling stresses at the upper and the lower skin–core interfaces, vs the external load relative to the critical load panel. The values of the unsymmetrical mode are always smaller than those corresponding to the symmetrical mode, since the critical mode shape of the panel is symmetric. The vertical deflections, see Fig. 7(a) correspond to the imperfection patterns, thus in case of a symmetrical imperfection mode it yields $w_t = -w_b$ that increase as the load ratio approach 1 and $w_t = w_b$ in case of an unsymmetrical imperfection mode. The bending moments, see Fig. 7(b) follow the same trends as those of the deflection, but their increase is lower as compared to the deflections. When the imperfection pattern is symmetrical, the shear stress resultant in the skin-panels, see Fig. 7(c) are symmetrical, while those of the core are null. In case of an unsymmetrical imperfection mode the shear resultants are mainly carried by the core and the shear resultants in the skins are very small. High peeling stresses at the skin–core interfaces that are usually the major cause of a premature failure in the form of debonding of the skin-panel from the adjacent core, appear in Fig. 7(d). These stresses increase at a very steep gradient when the load ratios approach 1 and the imperfection pattern is symmetric. In this case the peeling stresses at the upper and lower skin–core interfaces are identical. The stresses are much lower when the imperfection pattern is unsymmetric and with opposite signs. It should be noticed that the peeling stress capacity of a flexible core is usually very low, thus moderate values of peeling stress may initiate debonding that leads to premature catastrophic failure of such panels.

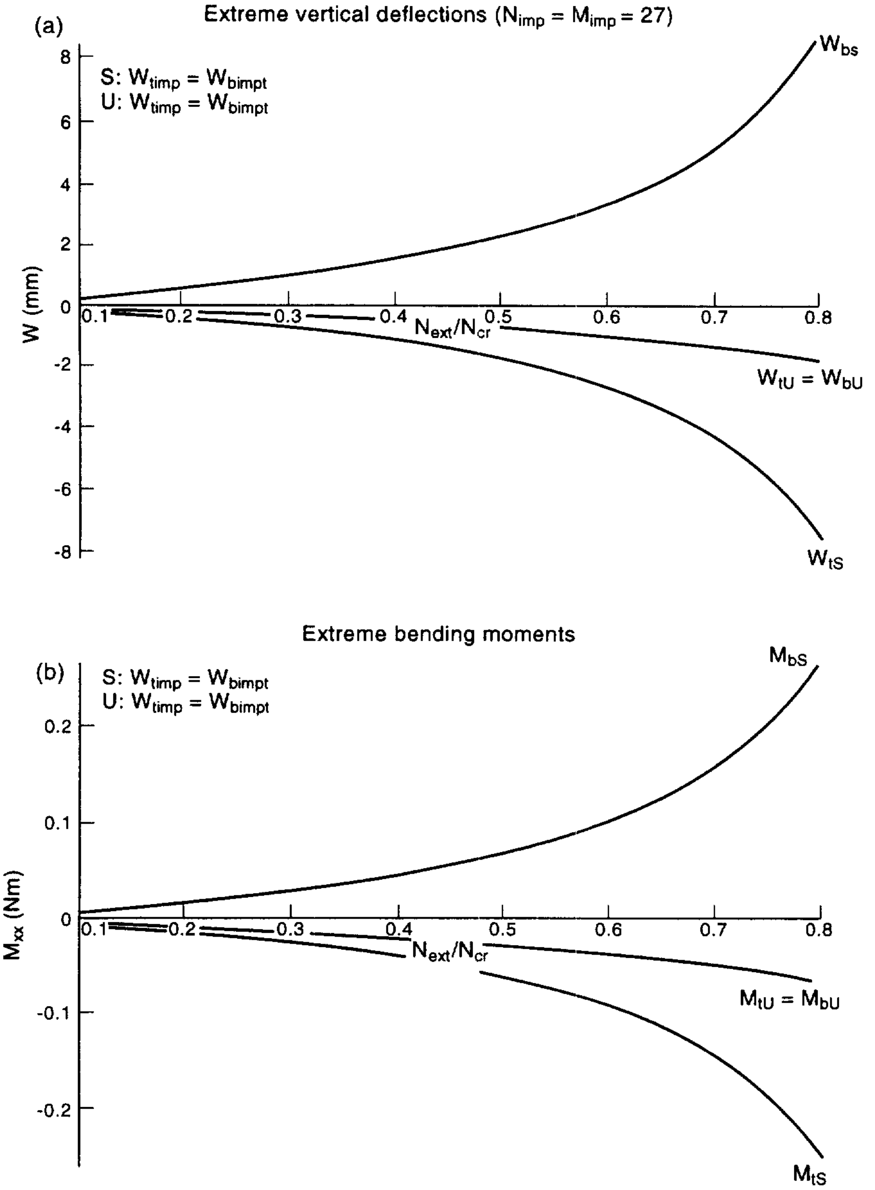

Fig. 7. Symmetrical and unsymmetrical imperfection results with $w_{jimp} = h/10$ ($j = t, b$) of a typical panel: (a) vertical deflections; (b) bending moments of upper and lower skin-panels; (c) shear resultants of skin-panels and core; and (d) peeling stresses at upper and lower skin–core interfaces.

SUMMARY AND CONCLUSIONS

A rigorous systematic stability analysis of sandwich panels with a flexible core that uses high-order theory is presented. The formulation uses a variational procedure along with kinematic relations, based on small deflections and moderate rotations, to derive the governing equations with the appropriate boundary conditions for the various stages of buckling. Pre-buckling and buckling governing equations and the corresponding boundary conditions have been derived using a perturbation technique. Imperfection analysis equations have been defined and is merely used to determine the magnitude of the stresses involved.

The high-order analysis is general, applicable to any type of core, isotropic or orthotropic, to any type of skin-panels, isotropic, orthotropic or composite laminated with non-identical skins. The formulation also allows non-identical boundary conditions at the various skin-panels edges, which is required when sandwich panels with a "soft" core that are supported only at the lower skin-panel are considered.

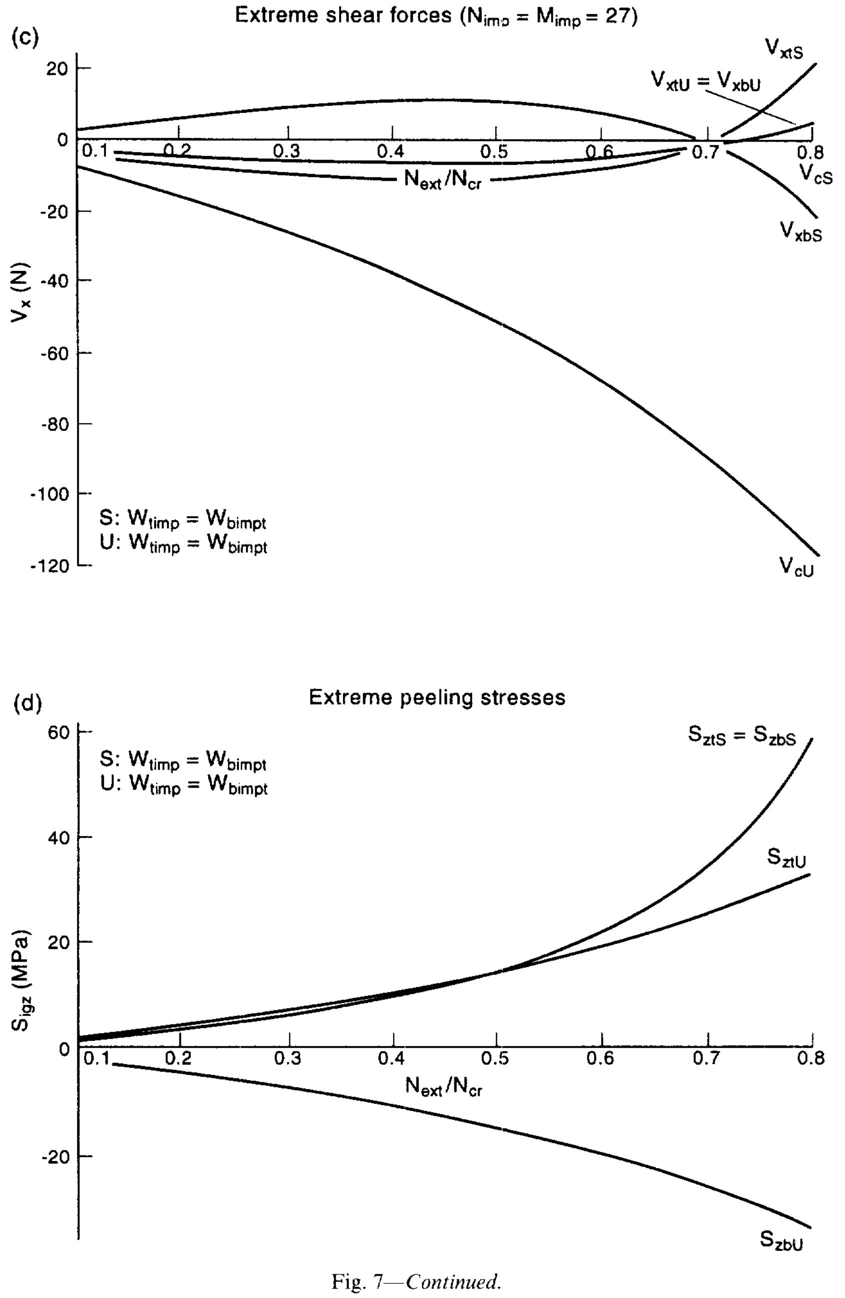

Fig. 7—*Continued.*

Closed form solutions of the prebuckling stage that are independent of the coordinates exist in cases of isotropic or orthotropic skin-panels with movable in-plane boundary conditions, and for a general symmetric composite laminated skin-panels in case of immovable boundaries. The bifurcation loads are determined through the solution of the buckling stage governing equations. Closed form solutions for various identical classical boundary conditions at the same edge exist. However, for simplicity and brevity only the equations, stiffness and geometrical matrices of a simply-supported panel are presented. The geometrical matrix is a diagonal matrix with two non-zero terms. Thus, only two eigenvalues and two eigenvectors (mode shapes) exist for every mode number. In the case of identical skin-panels the corresponding buckling mode shapes consist of global (asymmetrical with respect to center of core) and local (symmetrical) modes. In the case of a "stiff" core the global mode shape with low mode number governs while the "soft" cores the critical load usually correspond to the local (wrinkling) mode shape. In the case of unidentical skin-panels the buckling mode shapes are neither symmetric nor asymmetric and a separation to symmetrical and non-symmetrical mode is artificial and not beneficial. The classical approaches, that defines buckling through global buckling or wrinkling buckling separately, yields conservative results in case of sandwich panels with a "soft" core.

The imperfection analysis is presented to predict quantitatively the buckling behavior in terms of deformations, stresses and internal resultants at various load levels of the external compressive loads. The imperfection analysis study reveals that the peeling stresses increase at a very steep gradient as the external inplane load approach the critical one thus leading to high peeling stresses even at low load ratio. These stresses may exceed the stress allowable of the skin–core interface and might initiate debonding which usually leads to a premature failure in the form of debonding of one of the skin-panels from the core.

The formulation presented herein enhances the physical insight of the buckling behavior. It allows the investigation of the panels with unidentical boundary conditions at the upper and the lower skin-panel edges. It defines the governing equations for the pre-buckling and buckling stages and determines the critical load and the corresponding mode shape of panels with a rigid or a "soft" core. The immature collapse associated with buckling tests of such structures is explained through the imperfection analysis. The use of the high-order theory along with the presented analysis is recommended whenever sandwich panels with an out of plane flexible core are concerned.

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APPENDIX A

Stiffness matrix—coefficients

$$K_{11mn} = -\frac{A_{11t}m^2\pi^2}{a^2} - \frac{A_{66t}n^2\pi^2}{b^2} \quad K_{12mn} = -\frac{A_{66t}mn\pi^2}{ab} - \frac{A_{12t}mn\pi^2}{ab} \quad K_{17mn} = 1$$

$$K_{22mn} = -\frac{A_{22t}n^2\pi^2}{b^2} - \frac{A_{66t}m^2\pi^2}{a^2} \quad K_{28mn} = 1$$

$$K_{33mn} = -\frac{A_{11b}m^2\pi^2}{a^2} - \frac{A_{66b}n^2\pi^2}{b^2} \quad K_{34mn} = -\frac{A_{66b}mn\pi^2}{ab} - \frac{A_{12b}mn\pi^2}{ab} \quad K_{37mn} = -1$$

$$K_{44mn} = -\frac{A_{66b}m^2\pi^2}{a^2} - \frac{A_{22b}n^2\pi^2}{b^2} \quad K_{28mn} = -1$$

$$K_{55mn} = -\frac{D_{11t}m^4\pi^4}{a^4} - 2\frac{m^2n^2\pi^4}{a^2b^2}(2D_{66t} + D_{12t}) - \frac{D_{22t}n^4\pi^4}{b^4} - \frac{E_c}{c} \quad K_{56mn} = \frac{E_c}{c}$$

$$K_{57mn} = -\frac{m\pi(d_t + c)}{2a} \quad K_{58mn} = -\frac{n\pi(d_t + c)}{2b}$$

$$K_{66mn} = -\frac{D_{11b}m^4\pi^4}{a^4} - 2\frac{m^2n^2\pi^4}{a^2b^2}(2D_{66b} + D_{12b}) - \frac{D_{22b}n^4\pi^4}{b^4} - \frac{E_c}{c}$$

$$K_{67mn} = \frac{m\pi(d_b + c)}{2a} \quad K_{68mn} = -\frac{n\pi(d_b + c)}{2b}$$

$$K_{77mn} = \frac{c}{G_{cxr}} + \frac{m^2\pi^2c^3}{12E_{cv}a^2} \quad K_{78mn} = \frac{mn\pi^2c^3}{12E_{cr}ab}$$

$$K_{88mn} = \frac{c}{G_{cyr}} + \frac{n^2\pi^2c^3}{12E_{cr}b^2}.$$